ABSTRACT:

On finite hyperelasticity using stretch tensors

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In the field of phenomenological modelling of isotropic and incompressible hyperelastic materials in continuum mechanics, constitutive models of finite elasticity formulated with the left or the right Cauchy-Green tensor as arguments are frequently applied. The best-known example of this approach is the generalized Mooney-Rivlin model. As we know, both the left and the right Cauchy-Green tensors are already pronounced nonlinear deformation measures. Even the simplest neo-Hookean model with only one material parameter, whose Cauchy stress tensor depends linearly on the left Cauchy-Green tensor, shows pronounced non-linear behaviour.
In the presentation, the right and left stretch tensors are used as intrinsic arguments of the Helmholtz free energy density because these deformation measures depend rather linearly on the deformation. To formulate such a model compatible with the Clausius-Duhem inequality and to derive a hyperelastic potential relation, the polar decomposition of the deformation gradient is considered and the stress power is reformulated using the co-rotated Kirchhoff stress tensor and an associated deformation rate. The evaluation of the Clausius-Duhem inequality yields the potential relation for the co-rotated Kirchhoff stress. A transformation yields the Cauchy stress as function of the left stretch tensor. Based on this outcome, some simple and more complicated energy functions are evaluated for uniaxial tension/compression, simple and pure shear as well as biaxial tension. The results are compared with simulations based on generalised Mooney-Rivlin type models and discussed.